Evaluation of Spectra of Baryons Containing Two Heavy Quarks in Bag Model

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Abstract

In this work, we evaluate the energy spectra of baryons which consist of two heavy and one light quarks in the MIT bag model. The two heavy quarks constitute a heavy scalar or axial vector diquark. Concretely, we calculate the spectra of $|q(QQ')>_{1/2}$ and $|q(QQ')>_{3/2}$ where Q and Q' stand for b and/or c quarks. Especially, for $|q(bc)>_{1/2}$ there can be a mixing between $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ where the subscripts 0 and 1 refer to the spin state of the diquark (bc), the mixing is not calculable in the framework of quantum mechanics (QM) as the potential model is employed, but can be evaluated by the quantum field theory (QFT). Our numerical results indicate that the mixing is sieable

1 Introduction

At present, the non-perturbative QCD which dominates the low energy physics phenomena, is not fully understood yet, a systematic and reliable way for evaluating the non-perturbative QCD effects on such as the hadron spectra and hadronic matrix elements is lacking. Fortunately, however, for the heavy flavor mesons or baryons which at least contain one b or c quarks (antiquarks) the situation becomes simpler due to an extra $SU_f(2) \otimes SU_s(2)$ symmetry[1]. The studies in this field provide us with valuable information about the QCD interaction and its low energy behavior.

Among all the interesting subjects, the hadron spectra would be the first focus of attention. The spectra of the J/ψ and Υ families have been thoroughly investigated in different theoretical

approaches. Commonly, the spectra are evaluated in the potential model inspired by QCD, where the QCD Coulomb-type potential is directly derived from the one-gluon-exchange mechanism, and the confinement term originating from the non-perturbative QCD must be introduced by hand[2]. For the heavy quarkonia where only heavy quark flavors are involved, the potential model definitely sets a good theoretical framework for describing such systems where relativistic effects are small compared to the mass scale. An alternative model, the bag model can also provide a reasonable confinement for quarks. In fact, for light hadrons, especially light baryons, the bag model may be a better framework for describing their static behaviors. The MIT bag model has some advantages [3]. First, even though it does not hold a translational invariance, the quarks inside the hadron bag obey the relativistic Dirac equation and moreover, they can be described in the Quantum Field Theory (QFT) framework, namely there exist creation and annihilation operators for the constituents of the hadron. The latter property is very important for this work, that we can calculate a mixing between $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ (the notations will be explained below) states and it is impossible in the Quantum Mechanics (QM) framework.

Another interesting subject is if the diquark structure which consists of two quarks and resides in a color-anti-triplet $\bar{3}$, exists in baryons. Its existence, in fact, is till in dispute. For light diquark which is composed of two light quarks, the relativistic effects are serious and the bound state should be loose. By contraries, two heavy quarks (b and c) can constitute a stable bound state of $\bar{3}$, namely a diquark which serves as a source of static color field[4]. As a matter of fact, the un-penetrable bag boundary which provides the confinement conditions to the constituents of the hadron, is due to the long-distance non-perturbative QCD effects, to evaluate the spectra, one needs to include the short-distance interaction between the constituents and it can be calculated in the framework of the perturbative QCD. In this work, we are going to evaluate the spectra of baryons which contains two heavy quarks (b and/or c) and a light quark and take the light-quark-heavy-diqaurk picture which obviously is reasonable for the case of concern.

For evaluating the hadron spectra, the traditional method is the potential model. For baryons, the quark-diquark picture can reduce the three-body problem into a two-body problem and leads to a normal Schrödinger equation. Solving the Schrödinger equation, one can get the binding energy of quark and diquark[5, 6]. In recent years, remarkable progresses have been made along this direction. The authors of refs. [7] have carefully studied the short-distance and long-distance effects, then derived a modified potential and obtained the spectroscopy of the baryons which contain two heavy quarks by using the non-relastivistic Schrödinger equation. Meantime, in Ebert et al.'s new work, the light quark is treated as a fully relativistic object and the potential is somehow different from that in their earlier work[8]. In their works, not only the ground states of such baryons are obtained, but also the excited states are evaluated.

However, the potential model has two obvious drawbacks. First, even though the diquark is heavy, the constituent quark mass of the light quark is still comparable to the linear momentum which is of order of Λ_{QCD}. Thus the reduced mass is not large and the relativistic effects are still significant. Secondly, working in the framework of QM, it is impossible to estimate the mixing of $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ where the subscripts 0 and 1 of the (bc)-diquark denote the total spin of the subsystem, i.e. the (bc)-diquark (we only consider the ground state of $l = 0$). The reason is that there are no creation and annihilation operators in the traditional QM framework, so the transition $(bc)_1 + q \to (bc)_0 + q$, i.e. $A + q \to S + q$ where the notation A and S refer

to the axial-vector and scalar diquarks respectively, is forbidden, even though the transition is calculable in QFT.

On other side, the MIT bag model does not suffer from the two drawbacks. In this picture, since the diquark is heavy, it hardly moves so that can be supposed to sit at the center of the bag, whereas the light quark is freely moving in the bag and its equation of motion is the relativistic Dirac equation with a certain boundary condition[3] and both the quark and diquark are quantized in the QFT. Thus the relativistic effects are automatically included. Secondly, one can deal with a possible conversion of the constituents in the bag in terms of QFT, namely one can let a constituent be created or annihilated, thus the transition $A+q \to S+q$ is allowed and the corresponding mixing of $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ is calculable.

Usually the bag model is not very applicable to the light mesons because the spherical boundary is not a good approximation for the two-body system. Even though the quark-diquark structure is a two-body system, the aforementioned problem does not exist because the diquark is much heavier than the light quark. The picture is in analog to the solar system or an atom where only one valence electron around the heavy nucleus, and spherical boundary would be a reasonable choice.

In this work, following the literature [3], we treat the short-distance QCD interaction between the light quark and heavy diquark perturbatively. Since the interaction energy $E_{int}(R)$ is not diagonal for $|q(bc)_1>_{1/2}$ and $|q(bc)_0>_{1/2}$, we may diagonalize the matrix to obtain the eigenvalues and eigenfunctions which would be the masses of the baryons with flavor $q(bc)$ and spin 1/2. Moreover, for the other baryons $|q(bb)_1>_{1/2(3/2)}$ $|q(cc)_1>_{1/2(3/2)}$ $|q(bc)_1>_{3/2}$, the diquark must be an axial vector due to the Pauli principle[9].

The paper is organized as follows, after the introduction, we derive all the formulation of $E_{int}(R)$ and M_B in Sec.II, then in Sec.III we present the numerical results and all concerned parameters, finally the last section is devoted to discussions.

2 Formulation

1. A brief review of the MIT bag model

The wavefunction of a light quark in the MIT bag obeys the Dirac equation for free fermion and a boundary condition which forbids the quark current to penetrate the bag boundary. It has a form[3]

$$q(\mathbf{r},\mathbf{t}) = \frac{\mathbf{N}(\chi)}{\sqrt{\mathbf{4}\pi}} \begin{pmatrix} (\frac{\omega+m}{\omega})^{1/2} i j_0(\chi r/R) U \\ -(\frac{\omega-m}{\omega})^{1/2} j_1(\chi r/R) \boldsymbol{\sigma}\cdot\mathbf{r} U, \end{pmatrix} \tag{1}$$

with

$$N^{-2}(\chi) = R^3 j_0^2(\chi) \frac{2\omega(\omega-1/R)+m/R}{\omega(\omega-m)}, \tag{2}$$

where j_l is a spherical Bessel function, U is a two-component Pauli spinor, the eigen-energy is

$$\omega(m,R) = \frac{[\chi^2+(mR)^2]^{1/2}}{R}, \tag{3}$$

and the eigenvalue χ satisfies an equation

$$\tan(\chi) = \frac{\chi}{1 - mR - [\chi^2 + (mR)^2]^{1/2}}. \tag{4}$$

In our picture, the two heavy quarks constitute a diquark which is a boson-like bound state of color $\bar{3}$ and because it is heavy, it hardly moves, but sits at the center of the bag. Its wavefunction can be written as[10]

$$\psi(r) = \frac{N}{4\pi} e^{\frac{\Lambda r^2}{2}} \quad \text{for the scalar diquark;} \tag{5}$$

$$\psi_\mu(r) = \frac{N}{4\pi} e^{\frac{\Lambda r^2}{2}} \eta_\mu \quad \text{for the axial vector diquark,} \tag{6}$$

where η_μ is the polarization vector which is normalized as $\eta^2 = -1$, Λ is a parameter and will be discussed later in the text.

2. Formulation for the baryon spectra.

In the CM frame of the baryon, the total mass of the baryon can be written as

$$M_B = M_D + \omega + E_{int}(R) + \frac{4}{3}\pi R^3 B - \frac{z}{R} \tag{7}$$

where $E_{int}(R)$ is the interaction energy between the diquark and light quark, B is the bag constant and z is a constant for the zero-point energy. ω and M_D are the eigen-energy of the free light quark and the mass of the heavy diquark which are given in the literature[3, 5]. In this work, following the standard procedure [11, 3], we are going to calculate the interaction energy $E_{int}(R)$. Generally, the interaction hamiltonian in the bag can be expressed as

$$H_{D'D} = -\frac{\lambda_1^a}{2}\frac{\lambda_2^a}{2} g_s^2 \int \overline{q}(\mathbf{x})\gamma^\mu q(\mathbf{x}) D_{\mu\nu} \Psi^*(\mathbf{y}) \frac{< D'|J^\nu|D >}{\sqrt{MM'}} \Psi(\mathbf{y}) d^3x d^3y. \tag{8}$$

It is noted that without mixing, i.e. $H_{D'D}$ is diagonal, $E_{int} = H_{D'D}$, however, if there is non-diagonal $H_{D'D}$, E_{int} is the eigenvalues of the hamiltonian matrix. The expectation value of the Casimir operator $< 0|\lambda_1^a\lambda_2^a|0 >= -16/3$, the strong coupling $g_s^2 = 4\pi\alpha_s$, $q(\mathbf{x})$ is the wavefunction of the free light quark, $< D'|J^\nu|D >$ is the effective vertex for $DD'g$ and $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge[12]. The form of such a propagator in the configuration space reads

$$G(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi}[\frac{1}{|\mathbf{r} - \mathbf{r}_0|} + \sum_{l=0}^{\infty} \frac{(l+1)(1-k)}{l + (l+1)k} \frac{(rr_0)^l}{R^{2l+1}} P_l(cos\theta)]. \tag{9}$$

In this work, $\mathbf{r_0}$ is small, thus the main contribution comes from the $l = 0$ component. Then, the expression can be simplified as:

$$G(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi}[\frac{1}{|\mathbf{r} - \mathbf{r}_0|} + \frac{\tilde{k}}{R}]. \tag{10}$$

It is noted that the term $\frac{\tilde{k}}{R}$ is due to the mirror charge effect[12], in fact, it indeed corresponds to the zero-point energy in the bag as Jaffe et al. suggested[3]. Actually, $\tilde{k}$ is related to the vacuum property and still serves as a free parameter that cannot be determined from any underlying theory yet. We choose $\tilde{k} = 0.87$ to fit the most recent lattice result about the spectrum of $(ccq)_{\frac{1}{2}^+}$.

3. The D'Dg effective vertex

If we only consider the ground states of the diquark, namely the two heavy quarks are in $l = 0$ color-anti-triplet state, the diquark can be either a scalar (denoted as S) with $s = 0$ or an axial vector (denoted as A) of $s = 1$. The effective vertices can be derived by the quantum field theory under the heavy quark limit[13]. The effective vertex for $SS'g$ is

$$< S'|J^\nu|S >= \sqrt{MM'}(f_1 v'^\nu + f_2 v^\nu) \tag{11}$$

and the $AA'g$ effective vertex is of the form

$$\begin{aligned} < A'|J^\nu|A > \quad = \quad & \sqrt{MM'}[f_3(\eta \cdot \eta'^*)v'^\nu + f_4(\eta'^* \cdot \eta)v^\nu + f_5(\eta \cdot v')(\eta'^* \cdot v)v'^\nu \\ & + f_6(\eta \cdot v')(\eta'^* \cdot v)v^\nu + f_7\eta'^{*\nu}(\eta \cdot v') + f_8(\eta'^* \cdot v)\eta^\nu] \end{aligned} \tag{12}$$

where $v,\ v',\ \eta_\mu$ and η'_μ are the four-velocities and polarization vectors of D and D' respectively, f_i's are the form factors at the vertices. As we did in our previous work [6] where the effective potential model was employed, for convenience of calculation, we would write the polarization into a spin-operator which acts on the wavefunction of the axial-vector diquark as

$$\eta_\mu = \frac{1}{\sqrt{2}}(\boldsymbol{\beta} \cdot \mathbf{s}, \mathbf{s}), \tag{13}$$

where higher order relativistic corrections proportional to $\frac{\mathbf{p}^2}{M_D^2}$ are neglected and the factor $1/\sqrt{2}$ is a normalization factor because $< s^2 >= s(s+1) = 2$. It is worth noticing that s_i and s_j do not commute with each other, thus one must be careful about their order in deriving formula.

The effective vertex for ASg is written as

$$\begin{aligned} < A'|J^\nu|S > \quad = \quad & \sqrt{MM'}[f_{11}\eta'^{*\nu} + f_{12}(\eta'^* \cdot v)v'^\nu \\ & + f_{13}(\eta'^* \cdot v)v^\nu + f_{14} i\epsilon^{\nu l\rho\sigma}\eta_l'^* v'_\rho v_\sigma]. \end{aligned} \tag{14}$$

Here we must stress that for convenience of calculation, we turn η_μ into the quantum spin-operator as evaluating the interacting energy for $|q(QQ') >$, but as pointed above, we cannot calculate the mixing between $|q(bc)_1 >_{1/2}$ and $|q(bc)_0 >_{1/2}$ in QM, but need to carry out the derivation in QFT instead. Then, we have to keep the polarization η_μ in a 4-vector form as

$$\eta^\pm_\mu = \mp\frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \ \text{ and } \ \eta^0_\mu = (0, 0, 0, 1)$$

. Because the diquark is very heavy and hardly moves, $|\boldsymbol{\beta}| \ll 1$, one can use the polarization vector in the reference frame where the diquark is at rest.

In the heavy quark limit, we have

$$f_1 = f_2 = f_7 = f_8 = -f_3 = -f_4 = f_{14} = 1$$
$$f_{11} = f_{12} = f_{13} = 0, \tag{15}$$

and for very heavy diquark the above approximation holds[13].

4. The interaction energy

To calculate the interaction energy, the basic formula is eq.(8). For the heavy diquark, $|\mathbf{p} \ll M_D$, thus the relativistic corrections proportional to and higher than $\frac{\mathbf{p}^2}{M_D^2}$ can be safely ignored.

Based on the approximations, one can easily obtain the interaction energies.

For the baryon where the diquark is a scalar of $\bar{3}$, the interaction energy between the light quark and the scalar diquark is corresponding to the transition matrix element of $< q, S|H_{eff}|q, S >$, and can be expressed as

$$\begin{aligned} H_{SS} &= -\frac{\lambda_1^a}{2}\frac{\lambda_2^a}{2}\int\int \overline{q}(\mathbf{x})\gamma^\mu q(\mathbf{x}) D_{\mu\nu}\Psi^*(\mathbf{y})[f_1 v'^\nu + f_2 v^\nu]\Psi(\mathbf{y}) d^3x d^3y \\ &= g_s^2 N\int_0^R [j_0^2(\frac{\chi r_x}{R}) + j_1^2(\frac{\chi r_x}{R})]\frac{2}{r_x} d^3x, \end{aligned} \tag{16}$$

where $\mathbf{x}$ is the spatial coordinate of the light quark, $\mathbf{y}$ is that of the heavy diquark, $\mathbf{r} = \mathbf{x} - \mathbf{y}$ is the relative coordinate of the quark and diquark.

For the baryon where the diquark is an axial vector, the matrix element $< q, A|H_{eff}|q, A >$ is written as formula of $H_{AA\frac{1}{2}}$:

$$\begin{aligned} H_{AA\frac{1}{2}} &= -\frac{\lambda_1^a}{2}\frac{\lambda_2^a}{2}\int\int \overline{q}(\mathbf{x})\gamma^\mu q(\mathbf{x}) D_{\mu\nu}\Psi^*(\mathbf{y})[f_3(\eta\cdot\eta'^*)v'^\nu + f_4(\eta'^*\cdot\eta)v^\nu \\ &\quad + f_5(\eta\cdot v')(\eta'^*\cdot v)v'^\nu + f_6(\eta\cdot v')(\eta'^*\cdot v)v^\nu + f_7\eta'^*(\eta\cdot v') \\ &\quad + f_8(\eta'^*\cdot v)\eta^\nu]\Psi(\mathbf{y}) d^3x d^3y \\ &= g_s^2 N\int_0^R [j_0^2(\frac{\chi r_x}{R}) + j_1^2(\frac{\chi r_x}{R})] d^3x \int (-\frac{1}{|\mathbf{r_x} - \mathbf{r_y}|} + \frac{k}{R})|\Psi(y)|^2 d^3y \\ &\quad + g_s^2\frac{C_{\alpha\beta}}{M}\int_0^R \overline{q_\alpha}(\frac{\chi r_x}{R})\gamma^i q_\beta(\frac{\chi r_x}{R}) d^3x \int (\mathbf{S}\times\mathbf{q})^i(-\frac{1}{|\mathbf{r_x} - \mathbf{r_y}|} + \frac{k}{R})|\Psi(\mathbf{y})|^2 d^3y \end{aligned} \tag{17}$$

where $C_{\alpha\beta}$ refers to the spin projections of the quarks in the baryon. The transitional momentum $\mathbf{q}$ will change into the form $-i\nabla$ in configurational space acting on the relative position, and we then get:

$$\begin{aligned} H_{AA\frac{1}{2}} &= g_s^2 N\int_0^R [j_0^2(\frac{\chi r_x}{R}) + j_1^2(\frac{\chi r_x}{R})] d^3x \int (-\frac{1}{|\mathbf{r_x} - \mathbf{r_y}|} + \frac{k}{R})|\Psi(y)|^2 d^3y \\ &\quad + g_s^2\frac{C_{\alpha\beta}}{M}\int_0^R \overline{q}_\alpha(\frac{\chi r_x}{R})\gamma^i q_\beta(\frac{\chi r_x}{R}) d^3x \int \frac{[\mathbf{s}\times(\mathbf{r_x} - \mathbf{r_y})]^i}{|\mathbf{r_x} - \mathbf{r_y}|^3}|\Psi(\mathbf{y})|^2 d^3y \end{aligned} \tag{18}$$

where $\mathbf{q}$ is the exchanged momentum between the quark and diquark, and in the configuration space of the bag, it is an operator acting only on the relative coordinate $\mathbf{r}$ as $-i\nabla_{\mathbf{r}}$ and

$$\int e^{i\mathbf{q}\cdot\mathbf{r}}\frac{\mathbf{q}}{|\mathbf{q}|^2}\frac{d^3q}{(2\pi)^3} = \frac{i\mathbf{r}}{r^3}. \tag{19}$$

We finally obtain an integral which must be carried out numerically if we take the form of $|\Psi(y)|^2$ which is treated as δ function into our consider, we will finally read:

$$\begin{aligned} H_{AA\frac{1}{2}} &= g_s^2 N \int_0^R [j_0^2(\frac{\chi r_x}{R}) + j_1^2(\frac{\chi r_x}{R})](-\frac{1}{r_x} + \frac{k}{R}) d^3x \\ &+ g_s^2 \frac{N'}{M} \int_0^R j_0(\frac{\chi r_x}{R}) j_1(\frac{\chi r_x}{R}) < s_1 m_1' s_2 m_2' |[\frac{(\sigma \cdot \mathbf{S})}{r_x^2} \\ &- \frac{(\sigma \cdot \mathbf{r_x})(\mathbf{S} \cdot \mathbf{r_x})}{r_x^4}]| s_1 m_1 s_2 m_2 > d^3x \end{aligned} \tag{20}$$

For the baryon of spin 3/2, which is composed of the light quark of spin 1/2 and an axial vector diquark of spin 1, the interaction energy is

$$\begin{aligned} H_{AA\frac{3}{2}} &= g_s^2 N \int_0^R [j_0^2(\frac{\chi r_x}{R}) + j_1^2(\frac{\chi r_x}{R})] \frac{2}{r_x} d^3x \\ &+ g_s^2 \frac{N'}{M} \int_0^R j_0(\frac{\chi r_x}{R}) j_1(\frac{\chi r_x}{R}) < \frac{1}{2}, \frac{1}{2}, 1, 1 |[\frac{(\sigma \cdot \mathbf{S})}{r_x^2} \\ &- \frac{(\sigma \cdot \mathbf{r_x})(\mathbf{S} \cdot \mathbf{r_x})}{r_x^4}]| \frac{1}{2}, \frac{1}{2}, 1, 1 > d^3x \end{aligned} \tag{21}$$

For a mixing hamiltonian which originates from the transition $S+q \to A+q$, the non-diagonal interaction element is

$$H_{AS} = \int\int \overline{q}(\mathbf{x}) \gamma^\mu q(\mathbf{x}) D_{\mu\nu} \Psi^*(\mathbf{y}) (i\epsilon^{\nu l \rho\sigma} \eta_l'^* v_\rho' v_\sigma) \Psi(\mathbf{y}) d^3x d^3y \tag{22}$$

After a straightforward integration, we obtain

$$\begin{aligned} H_{AS} &= g_s^2 \frac{N''}{M} \int_0^R j_0(\frac{\chi r_x}{R}) j_1(\frac{\chi r_x}{R}) < s_1 m_1' |[\frac{(\sigma \cdot \mathbf{S})}{r_x^2} \\ &- \frac{(\sigma \cdot \mathbf{r_x})(\mathbf{S} \cdot \mathbf{r_x})}{r_x^4}]| s_1 m_1 > d^3x \end{aligned} \tag{23}$$

It is noted that here η_μ remains as a four-vector, and it reduces into a three-vector $\boldsymbol{\eta}$ which is not an operator.

With the mixing between $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$, one can write the real eigenstates of $(qbc)_{1/2}$ and $(qbc)'_{1/2}$ as

$$|\frac{1}{2}, \frac{1}{2}> = C|\frac{1}{2}, \frac{1}{2}, 0, 0> + D(-\sqrt{\frac{1}{3}}|\frac{1}{2}, \frac{1}{2}, 1, 0> + \sqrt{\frac{2}{3}}|\frac{1}{2}, -\frac{1}{2}, 1, 1>), \tag{24}$$

where C and D are the coefficients to be determined. The interaction energy is

$$\begin{aligned} <\tfrac{1}{2}, \tfrac{1}{2}|H|\tfrac{1}{2}, \tfrac{1}{2}> &= C^2 < \frac{1}{2}, \frac{1}{2}, 0, 0|H|\frac{1}{2}, \frac{1}{2}, 0, 0> + D^2(\frac{1}{3} < \frac{1}{2}, \frac{1}{2}, 1, 0|H|\frac{1}{2}, \frac{1}{2}, 1, 0> \\ &+ \frac{2}{3} < \frac{1}{2}, -\frac{1}{2}, 1, 1|H|\frac{1}{2}, -\frac{1}{2}, 1, 1> - \frac{2\sqrt{2}}{3} < \frac{1}{2}, \frac{1}{2}, 1, 0|H|\frac{1}{2}, -\frac{1}{2}, 1, 1>) \end{aligned}$$

$$+CD(-\sqrt{\frac{1}{3}}<\frac{1}{2},\frac{1}{2},0,0|H|\frac{1}{2},\frac{1}{2},1,0>+\sqrt{\frac{2}{3}}<\frac{1}{2},\frac{1}{2},0,0|H|\frac{1}{2},-\frac{1}{2},1,1>)$$
$$+CD(-\sqrt{\frac{1}{3}}<\frac{1}{2},\frac{1}{2},1,0|H|\frac{1}{2},\frac{1}{2},0,0>$$
$$+\sqrt{\frac{2}{3}}<\frac{1}{2},-\frac{1}{2},1,1|H|\frac{1}{2},\frac{1}{2},0,0>), \tag{25}$$

which contains both diagonal and non-diagonal interaction elements. It is easy to set it into a matrix form as

$$H=\begin{pmatrix} H_{SS} \; H_{SA} \\ H_{AS} \; H_{AA} \end{pmatrix}, \tag{26}$$

and the corresponding Schrödinger equation is

$$\begin{pmatrix} H_{SS} \; H_{SA} \\ H_{AS} \; H_{AA} \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix}=E\begin{pmatrix} C \\ D \end{pmatrix}. \tag{27}$$

Diagonalizing the matrix, one can solve C and D which would determine the fraction of $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ in the eigenstates.

3 The numerical results

In this work, we have the input parameters as

$m_u=m_d=m_q\approx 0,\ m_s=0.279$ GeV,

$M_{cc}=3.26$ GeV, $M_{bb}=9.79$ GeV, $M_{bc}=6.52$ GeV [5].

α_s=0.23, $B=(0.145\text{GeV})^4$.

As argued above, the zero-point energy is not considered. Minimizing the expression of M_B in eq.(1) with respect to the bag radius R, we obtain an equation and then determine the R-value. Substituting the obtained R-value into the expressions, we achieve the following table. In the table, we list our numerical results for the baryons which contain two heavy quarks, meanwhile the results obtained in other approaches are presented in the table for a clear comparison.

$Notation$	$content$	J^p	M_B (our results)	M_B [5]	M_B [7]	M_B [8]	M_B [14]	M_B [15]	M_B [16]
Ξ_{cc}	(cc)q	$\frac{1}{2}^+$	3.55	3.66	3.48	3.620	3.55	3.66	3.61
Ξ^*_{cc}	(cc)q	$\frac{3}{2}^+$	3.59	3.81	3.61	3.727	3.64	3.74	3.68
Ω_{cc}	(cc)s	$\frac{1}{2}^+$	3.73	3.76	3.59	3.778	3.66	3.74	3.71
Ω^*_{cc}	(cc)s	$\frac{3}{2}^+$	3.77	3.89	3.73	3.872	3.73	3.82	3.76
Ξ_{bb}	(bb)q	$\frac{1}{2}^+$	10.10	10.23	10.09	10.202	-	10.34	-
Ξ^*_{bb}	(bb)q	$\frac{3}{2}^+$	10.11	10.28	10.11	10.237	-	10.37	-
Ω_{bb}	(bb)s	$\frac{1}{2}^+$	10.28	10.32	10.21	10.359	-	10.37	-
Ω^*_{bb}	(bb)s	$\frac{3}{2}^+$	10.29	10.36	10.26	10.389	-	10.40	-
Ξ_{cb}	(cb)q	$\frac{1}{2}^+$	6.80	6.95	6.82	6.933	-	7.04	-
Ξ'_{cb}	(cb)q	$\frac{1}{2}^+$	6.87	7.00	6.85	6.963	-	6.99	-
Ξ^*_{cb}	(cb)q	$\frac{3}{2}^+$	6.85	7.02	6.90	6.980	-	7.06	-
Ω_{cb}	(cb)s	$\frac{1}{2}^+$	6.98	7.05	6.93	7.088	-	7.09	-
Ω'_{cb}	(cb)s	$\frac{1}{2}^+$	7.05	7.09	6.97	7.116	-	7.06	-
Ω^*_{cb}	(cb)s	$\frac{3}{2}^+$	7.02	7.11	7.00	7.130	-	7.12	-

Table 1. The baryon spectra

4 Conclusion and Discussion

In this work, we evaluate the spectra of baryons which contains two heavy quarks in the MIT bag model. It is an approach parallel to the potential model which is widely adopted for studying heavy hadrons. One can notice from the table given in the text that the numerical results in the two approaches are consistent with each other by the order of magnitude, but there are obvious distinction in numbers.

The confinement in both the potential model and the bag model is artificially introduced which may reflect the non-perturbative QCD behavior in certain ways, but since none of them are derived from the first principle, one cannot expect them to be precise and it is reasonable that they result in different numbers which are related to the physics pictures and phenomenological parameters. As a matter of fact, in general the parameters are obtained by fitting data and while calculating the spectra of the lowest states of heavy quarkonia, various potential forms and sets of parameters can meet data. The MIT bag model is an alternative model and this work may be complimentary to the potential model.

No matter in the potential model or the bag model, there is a zero-point energy problem, which manifest the vacuum property of QCD and is not calculable at present. The zero-point energy would determine the exact positions of each baryon and its excited states, but dies not influence the relative distances between the states. In the bag model, situation is a bit different, because $E_0 \propto 1/R$, when to obtain M_B, we differentiate M_B with respect to R and the minimum determines the R-value. Since the energy of free light quark is $\sqrt{\chi^2 + (mr)^2}/R$ has the same form of E_0, thus its contribution can be attributed to the little shift of the quark mass. The gaps

between various states are not affected. In the future when data of the spectra are accumulated, one can come back to make more accurate adjustment of all the phenomenological parameters. In this work, we ignore the zero-point energy as argued above.

The advantages of using the bag model to evaluate the spectra are two-folds. First the light quark obeys the relativistic Dirac equation, secondly, one can use the QFT to evaluate the possible mixing between $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ which is impossible in the regular QM framework. In our case, when we calculate the relativistic corrections to the interaction between quark and diquark, we only keep the terms up to order $\mathbf{p}/M$, which results in splitting of $B_{1/2}$ and $B_{3/2}$ of the same flavor combination. Because the diquark is very heavy and $M \gg |\mathbf{p}|$, this approximation would not bring up significant changes.

As a conclusion, we evaluate the spectra of heavy baryons containing two heavy quarks in the MIT bag model. The results are qualitatively consistent with that obtained in the potential model, but numerically differ by a few percents. Moreover, we obtain a relatively large mixing between $|q(bc)_0>_{1/2}$ and $|q(bc)_1>_{1/2}$ which will be tested in the future measurements. Since there are no data available so far, we cannot fix a few parameters such as the zero-point energy, and one can definitely expect some deviations from the real values. Once the data are accumulated in the future, we can fin-tune the model and parameters. Definitely, the data will provide us with valuable information about the model and parameters.

Even though the present B-factories BaBar and Belle cannot produce such baryons, TEVATRON and LHC which will begin running in 2007, may measure the spectra of such baryons, especially the long-expected Next-Linear-Collider (NLC) will offer us an idealistic place for studying such hadron spectra and our knowledge on the hadron structure can be greatly enriched.

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